A New Keynesian Model with Staggered Price and Wage Setting under Learning

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October 31, 2011

Abstract

This paper provides a study of the implications for economic dynamics when the central bank sets its nominal interest rate target in response to variations in wage inflation. I provide results on the existence, uniqueness, and stability under learning of rational expectations equilibrium for alternative specifications of the manner in which monetary policy responds to economic shocks when nominal rigidities are present. Monopolistically competitive producers set prices via staggered price contracts, and households set nominal wages in the same fashion. In this setting, the conditions for determinacy and learnability of rational expectations equilibrium differ from a model where only prices are sticky. I find that when the central bank responds to wage and price inflation and to the output gap, a Taylor principle for wage and price inflation arises that is related to stability under learning dynamics. In other words, any reaction of the interest rate to wage inflation, in addition to the response to price inflation, helps to avoid instability under learning and indeterminacy.

Keywords: Learning; Monetary Policy; Nominal Wage and Price Rigidity; Expectational stability.

JEL classification: E52, E31, E43, E58

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1. Introduction

The New Keynesian model has become a workhorse for the study of monetary policy in recent years. In this model, private agents' behavior depends not only on current policy but also on the expected course of monetary policy. Monetary models typically assume that authorities adopt either linear feedback monetary rules (Taylor-type rules) or optimal monetary policy rules in an attempt to control the economy. However, the role of these rules in stabilizing the economy has been criticized because of their potential to induce indeterminacy or multiple equilibria. Rules that induce indeterminate equilibria are considered undesirable. If the policy design does not lead to determinate rational expectations equilibrium (REE) then agents are unable to coordinate on a particular equilibrium of the model. When the monetary authority follows an indeterminate policy rule, the system could be unexpectedly volatile and agents would be unable to coordinate on the equilibrium targeted by the central bank. Under indeterminacy there are multiple REE "and the economy need not settle on the desired REE," as addressed by Evans and Honkapohja (2003).

However, when equilibria -even if multiple and therefore indeterminate- are learnable, agents can coordinate on the equilibrium targeted by the central bank, and eventually coincide with the REE. Agents "learn" the equilibrium of the model by making forecasts based on recursive least squares techniques and the data obtained from the economy. Agents' forecasts are updated over time as more data become available. These forecasts replace rational expectations in the model. When considering learnable equilibria even with discrepancy between the agents' expectations and the expectations required to yield a determinate REE, the system will converge to the REE. Therefore, designing rules that lead to learnable equilibria is desirable. Evans and McGough (2005a) studied determinacy and learnability conditions as selection criteria. Bullard and Mitra (2002) derived determinacy and learnability conditions for monetary policy linear feedback rules, and Evans and Honkapohja (2003) for optimal rules.[1] Other literature studies[2] have examined them for open-economy models.[3]

Recent work has shown that staggering of nominal wage contracts is important

[1]They propose that central banks should adopt an optimal policy rule that includes both expectations and fundamentals to ensure determinacy and learnability of the REE.

[2]Authors include Llosa and Tuesta (2008), Bullard and Schaling (2009), Bullard and Singh (2006), Zanna (2006), and Wang (2006). These authors examine rules that respond to exchange rate movements.

[3]Moreover, extensions of the model that consider determinacy and E-stability of REE when long-term interest rates are included in the model are studied in McGough et al. (2005), and Kurozumi and Van Zandweghe (2008), and when capital is included in the model are examined in Duffy and Xiao (2007) and Pfajfar and Santoro (2007). An overview of recent literature on expectations, learning, and monetary policy is provided in Evans and Honkapohja (2009).

to give rise to the key frictions that render monetary policy non-neutral. In fact, Christiano et al. (2005) conclude that wage stickiness -not price stickiness- appears more important in explaining output and inflation dynamics. Models that consider only sticky prices and not sticky wages have been criticized for producing "too sharp a real-wage decline in response to a tightening of monetary policy" as addressed in Christiano et al. (1999). Christiano et al. (2005), Altig et al. (2004), and Smets and Wouters (2007) further conclude that impulse response functions after a monetary policy shock are best fit by the model with staggered wage contracts, close to the impulse responses obtained by fitting a vector autoregression (VAR)[4]. This explanation validates wage stickiness as an important factor in explaining real effects of monetary policy.

It is not obvious that an optimizing-agent model with staggered nominal wage setting in addition to staggered price setting would yield determinacy and expectational stability (E-stability) conditions similar to a model in which only prices are sticky. One reason is that the volatility of aggregate wage inflation induces inefficiencies in the distribution of employment across households (Erceg et al., 2000); they show that policymakers' welfare function can be expressed in terms of the variances of the output gap, price inflation, and wage inflation.

I study determinacy and learnability conditions within Erceg et al.'s (2000) better-fitting model, which includes not only price but also wage rigidities.[5] This paper represents a contribution with respect to Bullard and Mitra (2002), which studies the determinacy and E-stability properties of a standard New Keynesian model, that is -price rigidity but wage flexibility.- The authors conclude that the equilibria can be learnable when the central bank rises its interest rate instrument more than one for one with increases in inflation. This condition is referred to the "Taylor principle condition." This paper is also related to the work of Galí (2007, chapter 7) and Flaschel et al. (2008), who study the question of determinacy when prices and wages are rigid. Galí (2007), through a numerical experiment, finds that if the interest rate reacts more than one to one to contemporaneous price inflation or wage inflation, then the REE is determinate; Flaschel et al. (2008) confirm these results analytically. With respect to this literature, this paper shows the determinacy conditions under rules that respond to forecasts and lagged, wage, price, and output gap data. Forward looking rules well

[4]In particular, Christiano et al. (1999) analyze impulse responses to an unexpected interest rate reduction, and they finds a slightly pro-cyclical real wage movement. The explanation to this modest response of real wages is that there is slow wage adjustment to any given change in labor demand.

[5]Carlstrom and Fuerst (2007) analyze whether monetary policy should respond to asset prices in a model with price and wage stickiness. They evaluate this question from the point of view of equilibrium determinacy and conclude that if wages and prices are sticky, the equilibra are likely to be indeterminate when the central bank adjusts policy in response to asset price movements.

describe the actual behavior of policymakers in countries like Germany, Japan and the U.S. More importantly, this paper studies the E-stability properties induced by linear feedback policy rules of different kinds, and McCallum (2009) argues that this criterion is more important than determinacy.

The analysis views the short term interest rate as the instrument of monetary policy design. The policy-design problem lies in characterizing how the interest rate should respond to changes in wage and price inflation to induce a learnable equilibrium given that both prices and wages exhibit rigidities. Responding to wages is important because the behavior of wages and wage inflation provide information about the rate of core inflation, as in De Long (1997). When the policymaker strictly targets price inflation strictly in a model that includes staggered wage setting, there is a considerably large welfare loss due to substantial variation in the nominal wage inflation and the output gap. Erceg et al. (2000) further conclude that hybrid rules according to which the nominal interest rate responds to wage inflation or the output gap, in addition to price inflation, perform nearly as well as the optimal rule, thus making them desirable. This paper also relates to Mankiw and Reis (2003) whose work determines which inflation rate the central bank should target if it wishes to maximize economic stability. They find that maximum stability of economic activity should use a price index that gives substantial weight to the level of nominal wages.

The main result of this paper is that a Taylor principle condition for wage and price inflation emerges when the central bank responds to wage inflation, price inflation and the output gap. When the central bank adjusts its interest rates positively and more than one for one with changes in price and wage inflation above target, and positively in response to changes in output above its target, a "leaning against the wind" policy is followed. If agents do not have rational expectations and they form forecasts using least squares learning, then a "leaning against the wind" policy from the central bank pushes the equilibrium toward the REE. Thus, a leaning against the wind policy for a combination of wage and price inflation when agents form forecasts using least squares learning is closely linked to learnable equilibria. This result holds when interest rates respond to current data, forward-looking expectations, and lagged data. To conclude, the result presented here supports Erceg et al. (2000) and Mankiw and Reis (2003) in that any reaction of the interest rate to wage inflation, in addition to the response to price inflation, helps to induce a learnable equilibrium, thereby making it desirable.

I derive the determinacy and learnability conditions following Evans and Honkapohja (2001) for a New Keynesian model with wage and price stickiness, as in Woodford (2003). The model consists of a Phillips curve for wages, a Phillips curve for prices, a real wage identity, a dynamic IS equation, and a Taylor-type rule that responds to the output gap and to price and wage inflation. The rules that target price inflation and wage inflation, respectively are studied under a contemporaneous data specification, a lagged data specification, and under forward-looking expectations. This timing of the data has been justified in papers such as those by McCallum (1999) and Bullard and Mitra (2002).

2. The Environment

2.1. The Model

The structural equations of the supply side of the model are from Woodford (2003) (chapter 8, section 2.2), as follows:

$$\pi_t = \beta\ \widehat{E}_t\pi_{t+1} + \kappa_p(x_t + u_t) + \xi_p(w_t - w_t^n), \tag{1}$$

$$\pi_t^w = \beta\ \widehat{E}_t\pi_{t+1}^w + \kappa_w(x_t + u_t) + \xi_w(w_t^n - w_t), \tag{2}$$

$$w_t = w_{t-1} + \pi_t^w - \pi_t, \tag{3}$$

where $\kappa_p \equiv \xi_p\omega_p$ and $\kappa_w = \xi_w(\omega_w + \sigma^{-1})$, where $\xi_p = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p(1+\omega p\theta_p)}$ and $\xi_w = \frac{(1-\alpha_w)(1-\alpha_w\beta)}{\alpha_w(1+v\theta_w)}$,

Here π_t^w is nominal wage inflation, w_t is the log real wage, w_t^n represents exogenous variation in the natural real wage and $\widehat{E}$ represents (possibly nonrational) expectations. The terms ξ_p, ξ_w, κ_p, and κ_w are all positive. Prices and wages are adjusted a la Calvo, where $1 - \alpha_p$ $(1 - \alpha_w)$ is the time-independent probability that each of the prices (wages) is adjusted each period. The parameter ξ_p represents the sensitivity of goods-price inflation to changes in the average gap between marginal cost and current prices; it is smaller as prices are stickier (α_p). The parameter ξ_w indicates, the sensitivity of wage inflation to changes in the average gap between households "supply wage" (the marginal rate of substitution between labor supply and consumption) and current wages, and it is a function of the Calvo parameter that denotes wage stickiness in the economy (α_w). $\omega_p > 0$ represents the elasticity of supply wage with respect to the quantity supplied at a given wage, while $\omega_w > 0$ measures the elasticity of the supply wage with respect to the quantity produced, holding fixed households' marginal utility of income, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution. Equations (1) and (2) are Phillips curves for prices and wages. Equation (3) is an identity for the real wage ($w_t = W_t/P_t$) expressed in logs and was rearranged in this form to provide a law of motion for the log of nominal wages.

The dynamic IS-type equation is described by

$$x_t = \widehat{E}_t x_{t+1} - \sigma(i_t - \widehat{E}_t \pi_{t+1} - r_t^n) \tag{4}$$

, where i_t is the nominal interest rate. Monetary policy is represented by a Taylor-type rule that responds to price inflation, wage inflation, and the output gap. The monetary policy parameters are denoted by ψ_{π^w}, ψ_π, and ψ_x. The baseline specification is

$$i_t = \psi_\pi \pi_t + \psi_{\pi^w} \pi_t^w + \psi_x x_t. \tag{5}$$

This will be called the *contemporaneous data specification* because the policymakers respond to contemporaneous data in their policy rules, and only the private sector forms expectations about future values of endogenous variables. The model consists of equations (1) − (5).

My analysis encompasses two scenarios defined as follows: **Scenario 1** illustrates the learnable and/or determinate regions of the parameter spaces of the policy coefficients as a function of the output gap and the price inflation parameters, for fixed values of the wage inflation parameter (0.001, 0.5, 1, and 1.5). The value of 0.001 corresponds to a rule where the interest rate is responding for the most part to price inflation and the output gap. The values of 0.5 and 1 correspond to a rule with a moderate to aggressive response to wage inflation in addition to the response to the output gap and price inflation. In various estimations of a Taylor rule for the United States, where the short term interest rate responds to price inflation and the output gap, the coefficient for price inflation was 1.5 and for the output gap was 0.5. These parameter values were found to characterize U.S. policy between 1987 and 1992 as addressed in Woodford (2003). The parameter value of 1.5 was the largest value chosen because it is within the range of reasonable policy parameter values, although the price inflation policy coefficient has often been estimated to be greater than 1.5.

Scenario 2 represents the learnable and/or determinate regions of the parameter space as a function of the output gap and the wage inflation, for fixed values of ψ_π (0.001, 0.5, 1, and 1.5). Scenarios 1 and 2 were considered to determine whether the determinate and learnable regions of the parameter space vary as the response to an increase in wage or price inflation increases.

2.2. Alternative Policy Rule Specifications

In addition to the baseline specification, I explore the determinate and E-stable regions of the parameter space when policymakers respond to lagged data in the policy rule and possible nonrational expectations of the policy variables.

The central bank is more likely to respond to past data of the variables included in the policy function because contemporaneous data from the quarter in which they need to make policy decisions are rarely available. Specifically, the central bank has readily available data from the past quarter to which the interest rate should respond. For that reason, in my analysis I include a policy function that responds to last-quarter data of the output gap, price inflation, and wage inflation. The model now comprises equations (1)-(4), and equation (5) is replaced by

$$i_t = \psi_\pi \pi_{t-1} + \psi_{\pi^w} \pi^w_{t-1} + \psi_x x_{t-1}. \tag{6}$$

A third specification of the model deals with the simultaneity issue of the data in the policy function. This includes a policy function that responds to forecasts of the output gap, the price inflation rate and the wage inflation rate. This forward-looking policy function is represented by

$$i_t = \psi_\pi \widehat{E}_t \pi_{t+1} + \psi_{\pi^w} \widehat{E}_t \pi^w_{t+1} + \psi_x \widehat{E}_t x_{t+1}. \tag{7}$$

Equation (5) is replaced by equation (7). In rule (7) the central bank adjusts its interest rate in response to current forecasts of one quarter ahead output gap, wage inflation and price inflation.

2.3. Determinacy and the Taylor principle

Rational expectations are viewed as a two-sided equilibrium in which expectations influence the time path of the economy and the time path of the economy affects expectations. A model is said to be determinate if it has a unique REE. When the central bank designs monetary policy rules that induce indeterminacy, agents may be unable to coordinate on a particular equilibrium in the model. In this case, under the rational expectations hypothesis the equilibrium will not always be unique. The general conditions for determinacy are outlined below. Consider a general class of models:

$$y_t = \alpha + B E_t y_{t+1} + \kappa\ e_t \tag{8}$$

, where y_t is a vector of endogenous variables and B is a matrix of coefficients.

In order to yield determinacy, the number of free variables in the model needs to be equal to the number of eigenvalues of matrix B with absolute value less than 1. Otherwise, the equilibrium is indeterminate. In the standard New Keynesian model with a linear feedback policy rule that responds to contemporaneous data, determinacy is governed by the Taylor principle. The Taylor principle concludes that the nominal interest rate should be adjusted more than one for one with changes in inflation. In that fashion, and as described in Bullard and Mitra (2002), as inflation increases the central bank should respond aggressively by increasing the real interest rate and thus reduce demand and inflationary pressures thereby inducing the economy back into the targeted equilibrium.

When the model yields indeterminacy there are multiple possible responses of the endogenous variables to shocks to fundamentals, some of which can create amplified economic fluctuations. However, endogenous variables can also respond to "sunspots" or extraneous random variables with no fundamental significance. Previous literature studies such as Evans and Honkapohja (2001) and Evans and McGough (2005b.) discuss the existence of models in which the solutions depends on sunspots. In "regular" cases, these solutions are explosive, but in "irregular" cases they are stationary. A regular linear model assumes that there exists a unique stationary REE. By contrast, in an irregular linear model, multiple stationary solutions are possible, particularly solutions that depend on sunspots. Changes in the variable (sunspot) could trigger self-fulfilling shifts in expectations and in the fundamentals in the model, creating disproportionately large fluctuations in the economy.

2.4. Learning Methodology

Under adaptive learning, it is typically assumed that agents do not know the structure of the model, but they try to estimate it using data from the economy. This approach assumes agents behave as econometricians by specifying an appropriate forecasting model or perceived law of motion (PLM) consistent with the REE of interest. In this paper, it is assumed that the agents are endowed with the correct specification of the model, the one associated with the fundamentals in the model or the MSV solution. The concept of E-stability is used to determine if the rational expectations equilibria are learnable. E-stability is a disequilibrium stability concept that determines whether an REE is stable under a reasonable learning rule (recursive least squares). Evans and Honkapohja (2001) have shown that E-stability governs the convergence of real-time recursive learning algorithms in many macroeconomic models. Therefore, a central bank's policy design is consistent with learnable equilibria, agents can coordinate on the equilibrium that the central bank targets and the equilibrium will eventually converge to the rational expectations dynamics as described in Bullard and Mitra (2002). For the E-stability condition, I assume that agents in the model no longer have rational expectations and instead use recursive least squares updating to form expectations.

E-stability under recursive least squares learning is locally convergent to the REE.

The general conditions for E-stability are outlined below. Following Evans and Honkapohja (2001) and considering a general class of models,

$$y_t = \alpha + BE_t y_{t+1} + \delta y_{t-1} + \kappa\ e_t \tag{9}$$

where y_t is an $n \times 1$ vector of endogenous variables, and B, δ, and κ are $n \times n$ matrices of coefficients. The MSV solutions take the form

$$y_t = a + by_{t-1} + ce_t, \tag{10}$$

with corresponding expectations

$$E_t y_{t+1} = (I + b)a + b^2 y_{t-1} + bce_t. \tag{11}$$

Inserting equation (11) into equation (9), it follows that the MSV solutions satisfy

$$(I - Bb - B)a = \alpha, \tag{12}$$

$$Bb^2 - b + \delta = 0, \tag{13}$$

$$(I - Bb)c = \kappa. \tag{14}$$

The actual law of motion (ALM) takes the form

$$y_t = \alpha + B(I + b)a + (Bb^2 + \delta)y_{t-1} + (Bbc + \kappa)e_t. \tag{15}$$

To determine E-stability, I consider equation (10) as the PLM and the mapping from PLM to ALM takes the form

$$T(a, b, c) = (\alpha + B(I + b)a, Bb^2 + \delta, Bbc + \kappa). \tag{16}$$

The expectational stability is determined by the following matrix differential equation:

$$\frac{d}{d\tau}(a, b, c) = T(a, b, c) - (a, b, c). \tag{17}$$

As in Evans and Honkapohja (2001), in order to analyze the local stability of system (17) at a rational expectations (RE) solution $\overline{a}$, $\overline{b}$ $\overline{c}$, the system is linearized at that RE solution. The E-stability conditions are governed by the equation for a, b, c in (15).

Using the rules for vectorization of matrix products, I compute

$$DT_a(\overline{a}, \overline{b}) = B(I + \overline{b}), \tag{18}$$

$$DT_b(\overline{b}) = \overline{b}' \otimes B + I \otimes B\overline{b}, \tag{19}$$

$$DT_c(\overline{b}, \overline{c}) = I \otimes B\overline{b}. \tag{20}$$

As in Bullard and Mitra (2002), a particular MSV solution $(\overline{a}, \overline{b}, \overline{c})$, is E-stable if the MSV fixed point of the differential equation (16) is locally asymptotically stable at that point. The following states the conditions for E-stability of the MSV solution $(\overline{a}, \overline{b}, \overline{c})$ and *is given at Evans and Honkapohja (2001, p.238)*:

Proposition 10.3: "Suppose the time-t information set is $(1,\ y_{t-1},\ e_t')$[6] an MSV solution $(\overline{a},\overline{b},\overline{c})$ to equation (8) is E-stable if all eigenvalues of the matrix $DT_a(\overline{a}, \overline{b})$, $DT_b(\overline{b})$, $DT_c(\overline{b}, \overline{c})$ given by equations (18), (19) and (20) have real parts less than 1. The solution is not E-stable if any of the eigenvalues have real parts larger than 1."

In other words, if the solution is E-stable, then agents are able to learn the equilibrium targeted by the central bank, and which will eventually converge to the REE.

2.5. Parameters

Analytic results are not tractable and so we proceed numerically as in Galí (2007), Evans and McGough (2007), and Bullard and Mitra (2007). The model was calibrated with parameter values from Amato and Laubach (2004).[7] They estimated impulse responses of wages and prices to a monetary policy shock. These parameters will be considered our baseline calibration. The robustness of results are also discussed under an alternative set of parameter values from Giannoni and Woodford (2003). The authors calibrate certain parameters and estimate others by minimizing the distance between predicted and estimated impulse responses. The parameter values obtained in their analysis take the values described below.

[6]The timing convention of the information set $(1,\ y_{t-1},\ e_t')$ is standard in the literature as suggested by Evans and Honkapohja (2009) review paper. It avoids simultaneity between expectations and outcomes

[7]The authors extend the analysis of Rotemberg and Woodford (1997) by adding a real wage series to a VAR.

Parameters	Amato and Laubach (2003)	Giannoni and Woodford (2003)
β	0.99	0.99
ω_p	0.33	0.33
ω_w	0.27	19.6
ξ_p	0.058	0.0022
ξ_w	0.066	0.0042
σ	1/0.26	1/0.16
ψ_π	$0.001, 0.5, 1, 1.5$	$0.001, 0.5, 1, 1.5$
ψ_{π^w}	$0.001, 0.5, 1, 1.5$	$0.001, 0.5, 1, 1.5$

Table 1: Parameter Values

3. Policy Rules under Determinate and Learnable Equilibria

3.1. Contemporaneous Data in the Policy Rule

3.1.1. Determinacy

The model can be simplified by substituting the policy rule (5) into (4), and writing the system involving the endogenous variables x_t, π_t,π_t^w,and w_{t-1} given by (1), (2), (3), and (4) in the following form:

$$y_t = \alpha + BE_t y_{t+1} + \kappa e_t \tag{21}$$

where y_t=[x_t,π_t,π_t^w, w_{t-1}]$'$, $\alpha = w_t^n$, e_t=[${r_t}^n$, u_t]$'$, and matrix B is defined in Appendix A.

In this setting x_t, π_t, and π_t^w are free variables. For that reason, three of the four eigenvalues of the system need to be inside the unit circle for determinacy; otherwise, the equilibrium is indeterminate. Figures 1 and 2 illustrate the results for Scenarios 1 and 2 respectively. Of note, the parameter space consistent with determinate equilibrium is similar to the one with E-stability for Scenarios 1 and 2 with contemporaneous data. The results are discussed in the next subsection.

3.1.2. Learning

I assume that y_t is not available when the forecasts $\widehat{E}_t y_{t+1}$ are formed, and the information set is represented by $(1, y_{t-1}, e_t')$. The model is written as

$$\widehat{y}_t = \alpha + \widehat{B} E_t \widehat{y}_{t+1} + \widehat{\delta} \widehat{y}_{t-1} + \widehat{\kappa}\ e_t, \tag{22}$$

where $\widehat{y}_t$=[x_t,π_t,π_t^w, w_t]$\prime$, $\alpha = w_t^n$, e_t=[$r_t{}^n$, u_t]$\prime$, and matrices $\widehat{B}$, $\widehat{\delta}$, and $\widehat{\kappa}$ are defined in Appendix A.

The MSV solution takes the form of equation (10). Equation (10) for $\bar{b}$ is characterized by a matrix quadratic that could have multiple solutions. The determinate equilibrium corresponds to the case where there is a unique solution for $\bar{b}$ with all of its eigenvalues inside the unit circle. E-stability of the MSV solution under learning is now considered. The PLM takes the form of the MSV solution. The mapping from PLM to ALM takes the form of equation (16). To compute the E-stability conditions I use derivatives (18), (19) and (20) where the three matrices require real parts less than 1 for E-stability. If at least one of the eigenvalues of the matrices has a real part greater than 1, then the equilibrium is E-unstable. After finding the MSV solution, E-stability conditions were numerically evaluated. These are depicted in Figure 1 for Scenario 1 and in Figure 2 for Scenario 2.

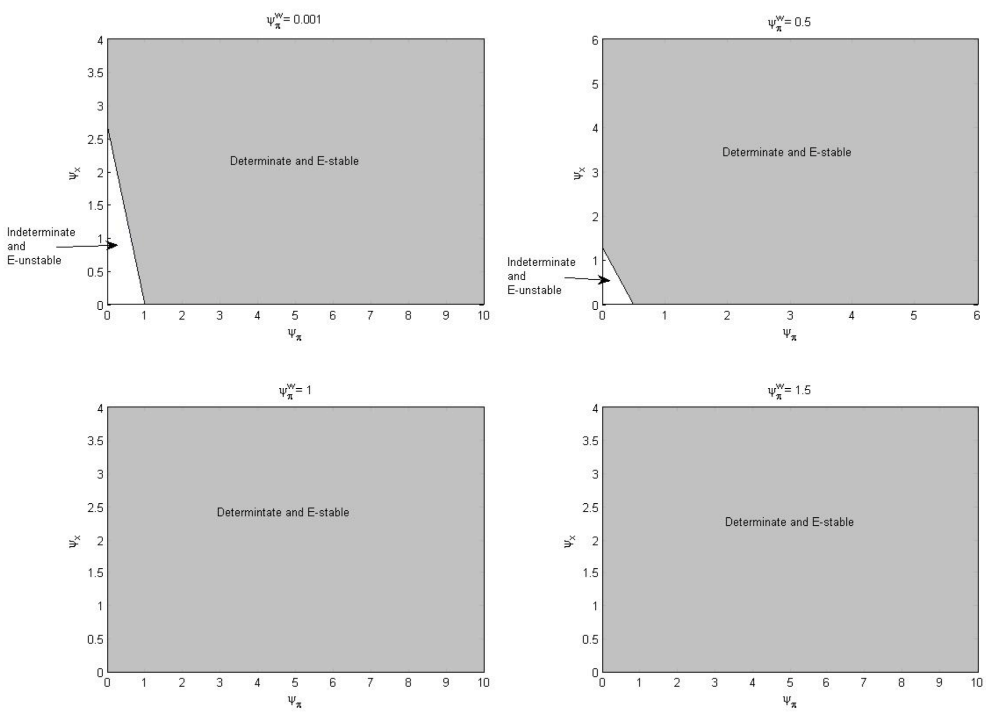

Figure 1: Determinacy and learnability for model with contemporaneous data in the policy rule. All parameters except for ψ_π and ψ_x are set at baseline values.

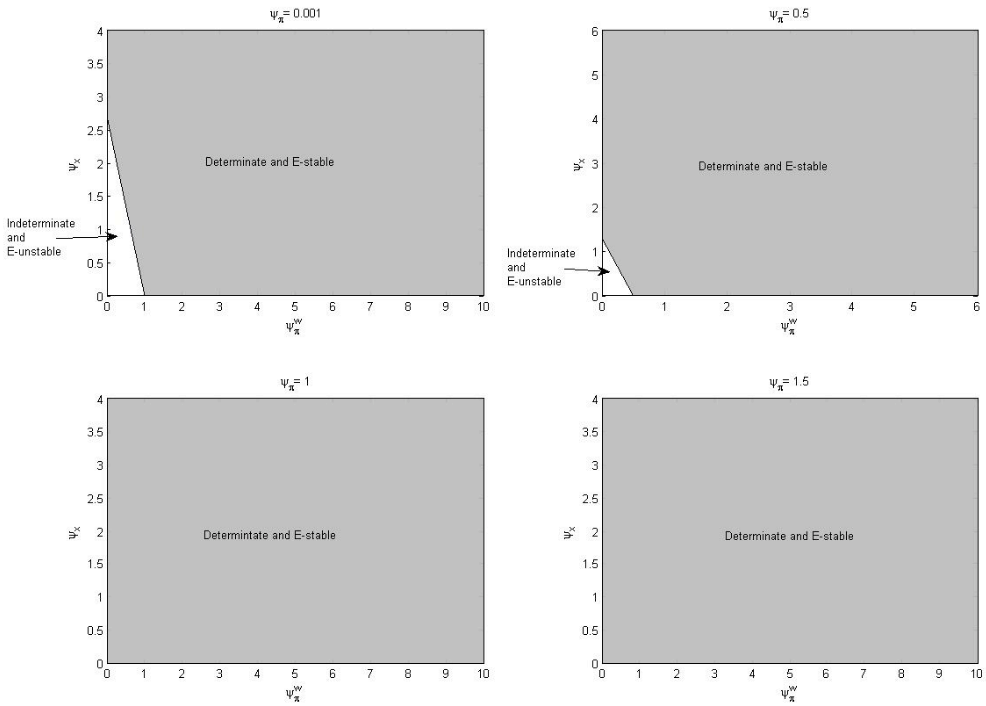

Figure 2: Determinacy and learnability for model with contemporaneous data in the policy rule. All parameters except for ψ_{π^w} and ψ_x are set at baseline values .

Determinacy and E-stability: Scenario One

The regions of determinacy and E-stability of the MSV solution as a function of ψ_π and ψ_x with all the other parameter values set at baseline values (Amato and Laubach, 2004), and where ψ_{π^w} takes values of 0.001, 0.5, 1 and, 1.5, are plotted in Figure 1. The plot where the interest rate rule responds to the output gap, price inflation, and an almost-negligible response to wage inflation shows a negatively sloped straight line that is consistent with the Taylor principle condition; this will be called Case 1. Points to the right of the line correspond to a determinate REE that are E-stable and points to the left are indeterminate and E-unstable. The line has a vertical intercept of $\psi_x = \frac{\kappa}{1-\beta} = 2.65$, where $\kappa = \frac{\sigma^{-1}+\omega}{\xi_p^{-1}+\xi_w^{-1}}$, and a horizontal intercept of $\psi_\pi = 1$. This result is analogous to that of Bullard and Mitra (2002) for contemporaneous data rules. The results and graphs obtained for the alternative calibration of Giannoni and Woodford (2003) are very similar to the graphs obtained for the baseline set of parameter values. Under the alternative calibration and when $\psi_{\pi^w} = 0.001$, the vertical axis finds its intercept at $\psi_x = 2.72$,[8] while the horizontal intercept is still at $\psi_\pi = 1$. As the response to wage inflation increases, the negatively sloped line consistent with the Taylor principle condition shifts toward the origin. Thus, when the response to wage inflation takes the value of ($\psi_{\pi^w} = 0.50$), the response to price inflation must also be of 0.50 in order to induce determinacy and E-stability. A Taylor principle condition arises, however, in this case the interest rate must respond more than one for one to a change in the average of price and wage inflations.

Previous estimations (Clarida et al., 2000) also suggest that κ_p could take higher values such as 0.30. This could be due in part to the presence of more flexible prices in the economy than what is suggested by our baseline calibration. A policy rule that responds to contemporaneous data has analogous implications for determinacy of REE and stability under learning regardless of the source of rigidity. As κ increases (due to more flexible prices or wages), the determinacy and E-stability line consistent with the Taylor principle condition pivots upward, finding its vertical intercept at $\psi_x = \frac{\kappa}{1-\beta}$. The determinate and E-stability region becomes smaller.

Determinacy and E-stability: Scenario Two

Figure 2 plots the determinacy and E-stability regions of the MSV solution as a function of ψ_{π^w} and ψ_x with all other parameter values set at baseline values, and where ψ_π takes values of 0.001, 0.5, 1 and 1.5. The determinate and E-stable region when $\psi_\pi = 0.001$ corresponds to an identical region to the first scenario. The negatively sloped line finds its horizontal intercept at $\psi_\pi = 1$ while the vertical intercept remains at $\psi_x = \frac{\kappa}{1-\beta} = 2.65$, where $\kappa = \frac{\sigma^{-1}+\omega}{\xi_p^{-1}+\xi_w^{-1}}$.

Equivalent results to Scenario 1 are obtained when the price inflation parameter is set at a moderate value of 0.50 and the wage inflation and the output gap are allowed to vary. In this case, the negatively sloped line that characterized the Taylor principle shifts toward the origin. These results are robust to our alternative calibration and are quite similar to those in Figure 2. As κ increase (as in Scenario 1) the line pivots upward with the horizontal intercept staying the same, making the determinate and E-stable region of the parameter space smaller.

To conclude, the model -given by equations (1)-(5)- is determinate and the unique REE is E-stable provided that $\psi_\pi + \psi_{\pi^w} > 1$. I drew values of ψ_π, ψ_{π^w}, and ψ_x from a uniform distribution [0,10]. I checked for determinacy and E-stability and recorded if the equilibrium was determinate and E-stable for values of $\psi_\pi + \psi_{\pi^w} > 1$. I repeated this exercise 1 million times; in 100 percent of the cases when $\psi_\pi + \psi_{\pi^w} > 1$, the

[8] Graphs for alternative calibration are available upon request.

equilibrium was determinate and E-stable. Thus a Taylor principle arises for wages and prices in a sticky wage and price model that governs determinacy and learnability of the REE. This result is robust to different calibrations of the structural parameters in the model.

3.2. Forward Expectations in the Policy Rule

3.2.1. Determinacy

The model with forward-looking expectations is composed of equations (1)-(4) and (7). The system of equations can be reduced to four equations when we substitute equation (7) into equation (4). As in the previous section, the endogenous variables in the model are x_t, π_t,π_t^w,and w_{t-1} and the reduced system is represented in the following form:

$$y_t = \alpha + BE_t y_{t+1} + \kappa e_t, \tag{23}$$

where y_t=[x_t,π_t,π_t^w, w_{t-1}]$\prime$, $\alpha = w_t^n$, e_t=[$r_t{}^n$, u_t]$\prime$, and matrix B is defined in Appendix B.

The variables x_t, π_t, and π_t^w are free variables; for that reason, three of the four eigenvalues of the system must be inside the unit circle for determinacy. Analytical results were not obtained in this case; thus, the findings were illustrated using a calibrated version of the model in Figures (3) and (4).

Determinacy Region: Scenario One

The region of determinacy of the MSV solution as a function of ψ_π and ψ_x, with all other parameter values set at baseline values, and when ψ_{π^w} takes values of 0.001, 0.5, 1, and 1.5 for the baseline calibration are plotted in Figure 3. For an almost-negligible response to the wage inflation ($\psi_{\pi^w} = 0.001$) in the policy rule (Case 1), the determinate equilibrium occurs when $\psi_\pi > 1$ and where ψ_x take values less than or approximately equal to 0.52. Therefore, in this case the Taylor principle condition is not enough to guarantee determinacy of the parameter space; there also must be a moderate response to the output gap. As the response to wage inflation becomes stronger, the negatively sloped line that divides the parameter space shifts toward the origin. Thus, an aggressive response to the combination of price and wage inflation ($\psi_\pi + \psi_{\pi^w} > 1$) and a modest response to the output gap lead to a determinate equilibrium.

Under the alternative calibration of Giannnoni and Woodford (2003) the MSV solution is determinate when ψ_x is roughly less than 0.25 and the combination of wage and price inflation is greater than 1 ($\psi_\pi + \psi_{\pi^w} > 1$). In addition, when one considers another calibration consistent with Clarida et al. (2000) where prices or wages are estimated to be more flexible than in the baseline calibration[9] the determinacy conditions change as follows: When $\xi_p = 1$, the horizontal line that divides the determinate and indeterminate regions of the parameter space pivots downward. Now, the equilibrium is only determinate for a combined response to wage and price inflation ($\psi_\pi + \psi_{\pi^w}$) greater than 1, a response to price inflation roughly less than 5, and a response to output gap less than 0.50. Moreover, as the response to inflation increases, the output response must decrease simultaneously to yield a determinate equilibrium. Therefore, for the alternative calibration, the Taylor principle condition is consistent with determinacy as long as the response to output is muted, and the response to inflation is not too aggressive.

Finally, when $\xi_w = 1$, the region of determinacy is found where the combined response of price and wage inflation is greater than 1, and as the response to wage inflation increases, the response to the output gap must decrease simultaneously. In other words, the horizontal line that divides the determinate and indeterminate regions of the parameter space shifts down. For example, when the response to wage inflation is equal to 1.5 the response to the output gap that guarantees determinacy must be less than 0.15.

Determinacy Region: Scenario Two

The region of determinacy of the MSV solution as a function of ψ_{π^w} and ψ_x with all other parameter values set at baseline values, and when ψ_π takes values of 0.001, 0.5, 1, and 1.5 is plotted in Figure 4. When policy rules respond to forward expectations of wage inflation and price inflation, the values assigned to ψ_x are crucial for determinacy. Under the baseline calibration, if the response of the policy rule to the output gap takes values roughly less than 0.51 and the combined response to price and wage inflation ($\psi_\pi + \psi_{\pi^w} > 1$) is greater than 1 then the REE is determinate. Under the alternative calibration (Giannoni and Woodford, 2003), the line that divides the parameter space into determinate and indeterminate regions pivots downward. Now, the MSV is determinate as long as $\psi_\pi + \psi_{\pi^w} > 1$, the response to wage inflation is less than 8, and the response to the output gap is less than 0.30. In addition, the response to the output gap must decrease to yield a determinate MSV solution as ψ_{π^w} increases. For example, when the combined response to wage and price inflation is equal to 1, the response to the output gap must be roughly less than 0.30 to yield

[9] Expressed as higher values of either ξ_p or ξ_w.

determinacy; however, when the response to wage inflation increases to 8, there must be zero response to the output gap.

Finally, I discuss the results for determinacy under more flexible prices and wages. In the case where $\xi_p = 1$, a Taylor principle condition for combined wages and prices yields determinacy as long as the response to the output gap is roughly less than 0.40. Under $\xi_w = 1$ the determinacy line pivots downward. In this case the equilibrium is determinate as long as the combined response to wage and price inflation is greater than 1, but the response to wage inflation is less than approximately 2.4. In addition, the response to output gap must be less than 0.40 and should be muted as the response to wage inflation increases. The extreme case is when the response to wage inflation is equal to 2.4 and the response to the output gap must be nil in order to yield a unique MSV solution.

I observe that values assigned to the policy parameter for the output gap (ψ_x) are important for REE determinacy. Only when the output gap policy parameter takes small values (roughly less than 0.5) is the equilibrium determinate. Therefore, in both scenarios, when we consider the baseline calibration the equilibrium is determinate as long as $\psi_\pi + \psi_{\pi^w} > 1$ and the response to ψ_x is modest. However, when the economy could have more flexible prices or wages, designing rules that respond aggressively to forecasts of price or wage inflation could lead to indeterminacy.

3.2.2. Learning

The E-stability conditions for rules with forward-looking expectations in the policy rule can be written as

$$\widehat{y}_t = \alpha + \widehat{B} E_t \widehat{y}_{t+1} + \widehat{\delta} \widehat{y}_{t-1} + \widehat{\kappa}\ e_t, \tag{24}$$

where $\widehat{y}_t$=$[x_t$,π_t,π_t^w, $w_t]\prime$, $\alpha = w_t^n$, e_t=$[r_t{}^n$, $u_t]\prime$, and matrices $\widehat{B}$, $\widehat{\delta}$, and $\widehat{\kappa}$ is defined in Appendix *B*.

The learnability conditions for rules that respond to forecasts have the same MSV solution (10), and the PLM of agents as in the case of the rules with contemporaneous data. In this case as well, the determinate equilibrium requires a unique solution for $\bar{b}$ with all eigenvalues inside the unit circle. After the MSV solution was found, I proceeded with the analysis in the same fashion as with contemporaneous data in the policy rule.

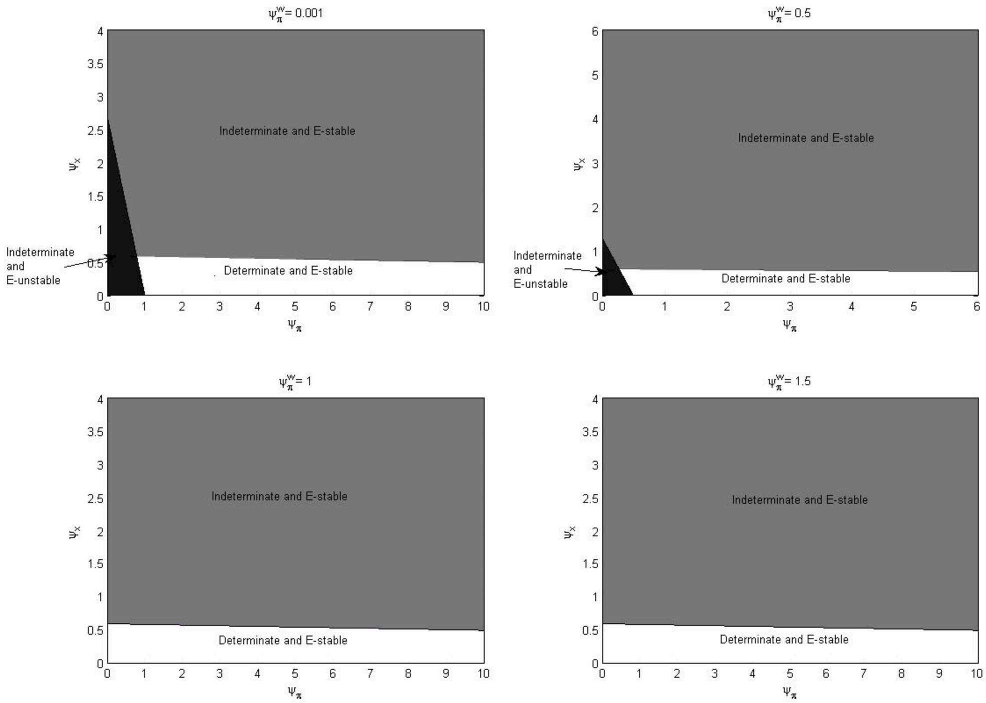

Figure 3: Determinacy and learnability for model with forward expectations in the policy rule. All parameters except for ψ_π and ψ_x are set at baseline values.

E-stability: Scenario One

Figure 3 plots the E-stable region of the MSV solution as a function of ψ_π and ψ_x with all other parameter values set at baseline values and with $\psi_{\pi^w} = 0.001, 0.5, 1,$ and 1.5. The E-stable region of the parameter space when $\psi_{\pi^w} = 0.001$ is characterized by the values to the right of the negatively sloped line with a horizontal intercept at $\psi_x = 1$ and vertical intercept at $\psi_x = \frac{\kappa}{1-\beta} = 2.65$. This result is in line with the Taylor principle condition. Therefore, the MSV solution is learnable even when found in the indeterminate region of the parameter space; the converse, however, is not true.

As the value of ψ_{π^w} increases to 0.5, the negatively sloped line shifts toward the origin. The average of price and wage inflation must be greater than 1 to induce E-stability. The alternative calibration yields results very similar to those in Figure (3). The Taylor principle condition for a combination of prices and wages still guarantees learnability of the MSV solution when forward-looking expectations are included in the feedback policy rule. In addition, when κ takes on a higher value, associated with flexible wages or prices, the E-stability line shifts upward, which reduces the the size

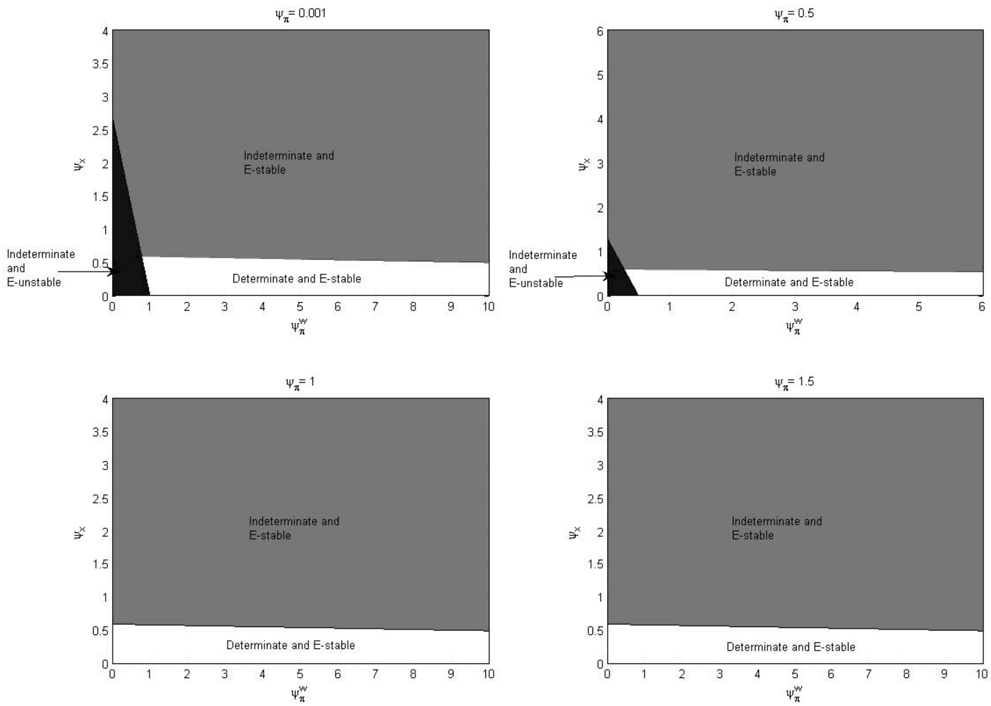

Figure 4: Determinacy and learnability for model with forward expectations in the policy rule. All parameters except for ψ_{π^w} and ψ_x are set at baseline values.

of the learnability region. As previously noted, the vertical intercept is determined by: $\psi_x = \frac{\kappa}{1-\beta}$.

E-stability: Scenario Two

The expectationally stable region of the MSV solution as a function of ψ_{π^w} and ψ_x with all other parameter values set at baseline values and $\psi_\pi = 0.001, 0.5, 1$ and 1.5 was plotted in Figure 4. This case resembles the E-stability region under scenario one. When $\psi_\pi = 0.001$ the horizontal intercept is $\psi_\pi^w = 1$ while the vertical intercept is found at ($\psi_x = 2.65$). When considering the alternate calibration the results closely resemble the Figure (4) with a slight difference in the vertical intercept. Points to the right of the line have an MSV solution that is learnable even if located in the indeterminate region of the parameter space. Analogous to scenario one, if the response to wage and price inflation is greater than 1 ($\psi_{\pi^w} + \psi_\pi > 1$), then the MSV solution is learnable. Therefore a Taylor principle condition arises for E-stability.

These results suggest that the E-stability regions of the parameter space for rules with price and wage inflation that exhibit contemporaneous data and forward-looking data for plausible sets of calibrations found in the literature are very similar. In both cases, a Taylor principle condition for wages and prices ($\psi_{\pi^w} + \psi_\pi > 1$) governs E-stability of the MSV solution. Furthermore, any response of the policy instrument to wage inflation, in addition to the conventional variables included in the policy reaction function, helps to achieve and preserve stability under learning. Therefore, it is desirable to design rules that use the policy instrument to respond to changes in price ***and*** wage inflation.

3.3. Lagged Data in the Policy Rule

3.3.1. Determinacy

As addressed by McCallum (1999), policymakers rarely have information about the variables included in the policy function in the period when they have to make decisions. For that reason, the use of lagged data in the policy function is a more plausible scenario. The model encompasses equations (1)-(4) and equation (6), where equation (6) has been moved one period forward. The system is rewritten involving the endogenous variables x_t, π_t,π_t^w, w_{t-1} and i_t in the following form:

$$y_t = \alpha + BE_t y_{t+1} + \kappa\, e_t \tag{25}$$

where y_t is a vector of endogenous variables and B is a matrix of coefficients. Matrix B is defined in Appendix C. It also contains an extended representation of the model.

The free variables in the model are x_t, π_t, and π_t^w, and the variables w_{t-1} and i_t are the predetermined variables. Thus, for uniqueness we should have three of the five eigenvalues inside the unit circle.[10] Figure 5 shows the results for Scenario 1 and Figure 6 for Scenario 2.

Determinacy: Scenario One

Figure 5 shows the determinacy regions of the MSV solution as a function of ψ_π, and ψ_x with all the other parameters set at baseline values, and where ψ_{π^w} takes values of 0.001, 0.5, 1, and 1.5. When there is an almost-negligible response from the central bank to lagged wage inflation ($\psi_{\pi^w} = 0.001$), a negatively sloped line with a horizontal intercept at $\psi_\pi = 1$ and a vertical intercept at $\psi_x = 2.65$ divides the parameter space. I observe two regions of determinacy: **Region 1** for values of ψ_π greater than 1 and a modest response to lagged output gap (less than 0.5), and **Region 2** for values to the

[10]See previous section on determinacy.

left of the negatively sloped line where ψ_π is at most 0.8 and ψ_x takes values roughly between 0.5 and 2.65. Determinacy in this setting posits a trade-off between (i) a relatively large response to lagged output gap and a relatively small response to lagged price inflation, or (ii) a relatively large response to lagged inflation (greater than 1) and a relatively small response to lagged output gap. Therefore, with lagged data in the policy rule the Taylor principle condition is neither necessary nor sufficient to guarantee uniqueness of equilibrium.

For the alternative calibration (Giannoni and Woodford, 2003), the determinacy regions are similar to the baseline calibration. Now the horizontal boundary that divides the explosive and determinate regions shifts downward, yielding a determinate equilibrium as long as ψ_x is roughly less than 0.25 and the combined response to wage and price inflation is greater than 1 (Region 1). Moreover, the negatively sloped line that divided the parameter space in case 1 shifts upward. The MSV solution is determinate as long as the combined response to inflation is less than 1 and the response to the output gap is roughly between 0.25 and 3 (Region 2). Under both calibrations, since the combined response to inflation increases and achieves a value greater than 1, the only determinate area that prevails is Region 1.

We now consider determinacy as $\xi_p = 1$. In this case, the horizontal line that divides the space between explosive and determinate pivots downward. The equilibrium is determinate when the combined response to wage and price inflation is greater than 1 but the response to price inflation is no greater than 5, and the response to output gap must be less than 0.6 (Region 1). In addition, in order to yield determinacy the output gap response must decrease as the price inflation response increases. Thus, when the price inflation response is equal to 5, the response to the output gap must be zero. The Region 2 is still present under more flexible prices but it is now larger. The determinate region can be found where the response to price inflation is roughly less than 1 and the response to the output gap is roughly between 0.5 and 5. As the combined response to inflation increases, we are left only with determinate Region 1. Lastly, under flexible wages, $\xi_w = 1$, determinate Region 2 is isomorphic to Region 2 under flexible prices. Moreover, Region 1 is determinate as long as $\psi_\pi + \psi_{\pi^w} > 1$ and the response to the output gap decreases as the response to wage inflation increases. For example, when the response to wage inflation is $\psi_{\pi^w} = 1.5$, ψ_x must be less than 0.15 to yield a determinate equilibrium. The horizontal line that divides the space between determinate and explosive shifts downward as the response to wage inflation increases.

Determinacy: Scenario Two

Figure 6 shows the determinacy regions of the MSV solution as a function of ψ_{π^w} and ψ_x with all the other parameters set at baseline values, and where ψ_π takes values of 0.001, 0.5, 0.1, and 1.5. The parameter space related to determinacy is almost the same as in Scenario 1. When $\psi_\pi = 0.001$, determinacy is characterized by (i) a region where the central bank responds strongly to lagged output gap and moderately to lagged wage inflation (**Region 2**) and (ii) a region where the interest rate responds more than one for one change in lagged wage inflation and the response to lagged output gap must be modest (**Region 1**). As $\psi_\pi = 0.5$, the line moves inward and now the condition that determines uniqueness of equilibrium is that $\psi_{\pi^w} + \psi_\pi > 1$ and a low response to the output gap. In addition, ψ_x could take values between 0.5 and 1.5 with a moderate response to ψ_{π^w} and yield determinacy.

For the alternative calibrations of Giannoni and Woodford (2003), Region 2, consistent with a low response to inflation, is larger and is found where the response to the output gap takes values between 0.3 and almost 3. Region 1 is somewhat different: Now we need not only a moderate response to output gap to yield determinacy, but the response to the wage inflation must be lower than approximately 7.5. Thus, the boundary between the determinate and explosive region pivots downward reducing the determinate region.

When one considers determinacy under $\xi_p = 1$, determinate Region 2 associated with a lower response to wage inflation and a response to output gap roughly between 0.60 and 5 is now larger. As the response to price inflation increases and becomes greater than 1, only determinate Region 1 prevails. Moreover, as the combined response to inflation increases, the horizontal line that divides the parameter space between determinate and explosive shifts downward; therefore as the response to inflation increases, the response to output must be increasingly modest for the MSV solution to be determinate.

Lastly, under $\xi_w = 1$, determinate Region 2 remains the same as under $\xi_p = 1$; however, Region 1 is different from what we observed. The horizontal line that divides the parameter space between determinate and explosive pivots downward. Therefore the region of determinacy can be observed where the combined response to inflation is greater than 1, but the response to wage inflation must be less than roughly 2.3; furthermore, the response to the output gap must be less than 0.5.

In summary, under flexible prices (flexible wages), Region 1 is determinate as long as the combined response to wage and price inflation is greater than 1, the response to output gap is decreasing as the response to price (wage) inflation increases, and as long as the response to price (wage) inflation is not too large (less than 5). Therefore, we have an additional condition for determinacy; as the central bank designs policy, its response to price or wage inflation should not be too aggressive in order to be consistent with determinacy.

The learnable region of the parameter space when lagged data are included in the policy rule is consistent with the determinate Region 1 discussed above. The results for learnability of the MSV solution follow.

3.3.2. Learning

The model was reshaped by substituting equation (6) into equation (4) and is written in terms of the endogenous variables x_t, π_t, and π_t^w, and w_t as follows:

$$\widehat{y}_t = \alpha + \widehat{B} E_t \widehat{y}_{t+1} + \widehat{\delta} \widehat{y}_{t-1} + \widehat{\kappa}\ e_t, \tag{26}$$

where $\widehat{y}_t$=$[x_t$,π_t,π_t^w, $w_t]\prime$, $\alpha = w_t^n$, e_t=$[{r_t}^n$, $u_t]\prime$, and see Appendix C for matrices $\widehat{B}$, $\widehat{\delta}$, and $\widehat{\kappa}$.

The learnability conditions for rules with lagged data have the same MSV solution (10), and the PLM of agents as for rules with contemporaneous data. The determinate case requires a unique solution for $\bar{b}$ with both eigenvalues inside the unit circle. After the MSV solution is found, the analysis follows the same steps as cases with contemporaneous data and forecasts in the policy rule. E-stability conditions were numerically evaluated and illustrated in Figures 5 and 6.

E-stability: Scenario One

Figure 5 shows the E-stable regions of the MSV solutions as a function of ψ_π and ψ_x with all the other parameters set at baseline values and where ψ_{π^w} takes values of 0.001, 0.5, 0.1. The MSV solution could be learnable if the central bank adjusts its interest rates more than one for one in response to changes in the the sum of price and wage inflation, as long as the response by the central bank to the output gap remains at low to moderate values of $\psi_x < 0.60$. The learnable region of the parameter space is roughly consistent with determinate Region 1 when the policy instrument responds to lagged data.

The alternative calibration of Giannoni and Woodford (2003) yields a determinate region as long as the combination of ψ_π and $\psi_{\pi w}$ is greater than 1, with a moderate response to ψ_x less than 0.25. Finally, when $\xi_p = 1$, the MSV is learnable as long as $\psi_\pi + \psi_{\pi^w} > 1$, the response to the output gap is roughly less than 0.60, and the

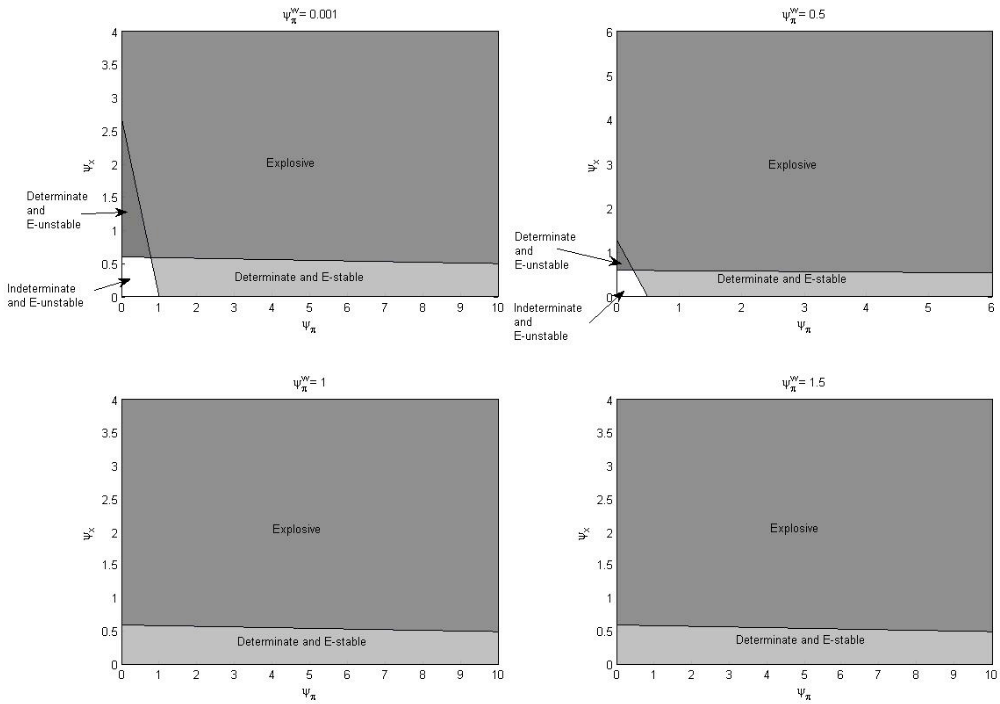

Figure 5: Determinacy and learnability for model with lagged data in the policy rule. All parameters except for ψ_π and ψ_x are set at baseline values.

response to price inflation is not too aggressive ($\psi_\pi < 5$). Moreover, as the response to price inflation increases the response to the output gap should decrease simultaneously. In the case where $\xi_w = 1$ equilibria are learnable when $\psi_\pi + \psi_{\pi^w} > 1$, combined with a moderate response to the output gap. As the response to wage inflation increases, the response to output gap should be decreasing.

E-stability: Scenario Two

Figure 6 shows the E-stable region of the MSV solutions as a function of ψ_{π^w} and ψ_x with all the other parameters set at baseline values, and where ψ_π takes values of 0.001, 0.5, 1 and 1.5. Analogous to scenario 1, in order to induce E-stability, the central bank must respond more than one for one to changes in the combination of lagged price and wage inflation and show a moderate response to the lagged output gap ($\psi_x < 0.5$).

Under the calibration of Giannoni and Woodford (2003), the parameter space is consistent with E-stability as long as the combined response to wage and price inflation is greater than 1 but no greater than approximately 7.5, and the response to the output gap must be modest and decrease simultaneously as the response to wage inflation

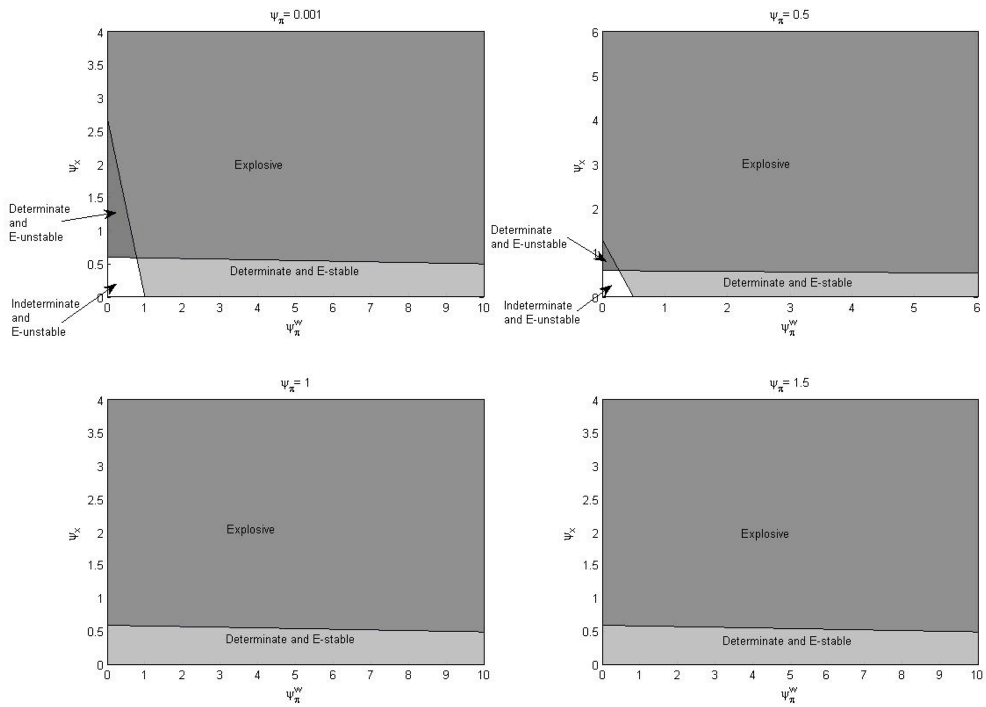

Figure 6: Determinacy and learnability for model with lagged data in the policy rule. All parameters except for ψ_{π^w} and ψ_x are set at baseline values.

increases. In addition, when $\xi_p = 1$ the combined response to inflation must be greater than 1 and the response to the output gap must be moderate ($\psi_x \leq 0.4$). Lastly, when I consider a calibration with $\xi_p = 1$, the combined response to price and wage inflation must be greater than 1 but the response to wage inflation cannot be large; $\psi_{\pi^w} \leq 2.3$ in order to induce learnability.

In a model with staggered wage and price contracts, if the central bank follows a rule that responds to lagged data, the Taylor principle condition induces a learnable MSV solution as long as the response to the output gap is moderate. An additional conditional arises, however, depending on the level of price and wage stickiness present in the economy. When considering prices (wages) that are more flexible than under the baseline calibration, in order to induce a learnable MSV solution, the response to price (wage) inflation must not be too aggressive, in some cases, the response must be less than 2.5. Thus, the size of the response to inflation matters in inducing a learnable MSV solution.

3.4. Policy Inertia

It has been widely addressed in the literature (Bullard and Mitra, 2007; Dennis, 2006; Cukierman, 1989; Brainard, 1967) the desirability for rules that respond cautiously to inadvertent changes in economic conditions. This caution can be modeled by having a central bank that responds to inertia on their policy rule. In this case, I discuss the results of determinacy and stability under learning using two variations of the monetary policy rules studied above. The first alternative policy rule is a lagged policy rule in which the central bank responds not only to lagged price and wage inflation and the output gap, but also includes a lagged interest rate term

$$i_t = \psi_\pi \pi_{t-1} + \psi_{\pi^w} \pi^w_{t-1} + \psi_x x_{t-1} + \psi_i i_{t-1}. \tag{27}$$

The second alternative policy rule sets the interest rate instrument in response to changes in the forecasts of price and wage inflation, the output gap, and the lagged interest rate. This forward- looking policy function is represented by

$$i_t = \psi_\pi \widehat{E}_t \pi_{t+1} + \psi_{\pi^w} \widehat{E}_t \pi^w_{t+1} + \psi_x \widehat{E}_t x_{t+1} + \psi_i i_{t-1}. \tag{28}$$

In these cases, the systems of equations are given by (1-4) and equation (27) for the first system (with lagged data), and equation (28) for the second system (with forecasts), and policy inertia in the monetary policy rule. The analyses of determinacy and E-stability of the MSV solutions are standard and follow sections 2.3 and 2.4.[11] As in Bullard and Mitra (2007), analytical solutions were not obtained for the E-stability conditions of stationary MSV solutions, however the results are discussed using the baseline calibration values of Amato and Laubach (2004).

Under the alternative policy rule that responds to lagged data and policy inertia (rule 1), the results compare to the graphs presented in Bullard and Mitra (2007) in the following way: Taylor-type rules that respond to policy inertia, where $\psi_i = 0.65$, with a calibration value justified in Bullard and Mitra (2007) have a more prominent determinacy and learnability area than in the case where no policy inertia is present. In fact, this area is even larger when the central bank responds to wage inflation in its policy instrument in a model in which both wage stickiness and price stickiness are considered. Moreover, when determinacy and learnability are considered under a pronounced value of policy inertia, $\psi_i = 5$, the area of interest appears to be larger than in Bullard and Mitra (2007). The prospect of rules that include policy inertia

[11] The matrices and graphs of results obtained when performing the determinacy and E-stability of stationary MSV solution analyses are available upon request.

of yielding a determinate equilibrium that is learnable is enhanced as the policymaker adjusts its instrument in response to changes in wage inflation. The same conclusion can be reached when considering rules that respond to forecasts of inflation and policy inertia; the desirable region is larger as wage inflation is included in the monetary policy rule.

Conclusion

The study of determinacy and stability under learning with various specifications of Taylor type-rules in a model with price rigidities was developed in Bullard and Mitra (2002). They conclude that the equilibria can be learnable when the central bank raises its interest rate instrument more than one for one with increases in inflation. This condition is referred to as the Taylor principle. Moreover, determinacy does not guarantee a learnable equilibrium. I build on this paper by considering the determinacy and learnability conditions in a model where monopolistically competitive firms, and households set prices and wages in staggered contracts following Erceg et al. (2000). Furthermore, I consider alternative specifications of a nominal interest rate rule followed by the central bank that responds not only to price inflation and the output gap but also to wage inflation. The main result is that when the central bank responds to wage and price inflation and the output gap, a Taylor principle for wage and price inflation arises: The nominal interest rate should be adjusted more than one for one with changes in (wage and price) inflation. This Taylor principle is closely linked with stability in the learning dynamics when the central bank adjusts its interest rates in response to current data and forward-looking expectations. Furthermore, when the central bank adjusts the interest rate in response to lagged data, the policy instrument must (i) respond moderately to changes in the output gap and (ii) meet the Taylor principle condition in order to yield stability under learning dynamics. Finally, in the case of lagged data, it is recommended that the policy response to wage and price inflation not be too aggressive, otherwise it could lead to instability of the MSV solution. In all cases, a reaction of the interest rate to wage inflation, in addition to the response to price inflation, could potentially help to enhance stability under learning. This is important because in considering learnable equilibria -even with discrepancy between the agents' expectations and the expectations required to yield a determinate REE- the system will converge to the REE. Results suggest that a central banker concerned with avoiding stability under learning should also respond to wage inflation in addition to price inflation. The desirability of having mixed rules that respond to wage and price inflation is also supported by Erceg et al. (2000) and Mankiw and Reis (2003).

Appendix

Appendix A:

$$B = g.\begin{pmatrix} 1 & \sigma - \beta\sigma\psi_\pi & -\beta\sigma\psi_{\pi^w} & -\xi_p\sigma\psi_\pi + \xi_w\sigma\psi_{\pi^w} \\ \kappa_p & \beta\left(1 + \kappa_p\sigma\psi_\pi + \sigma\psi_x + \kappa_w\sigma\psi_{\pi^w}\right) + \kappa_p\left(\sigma - \beta\sigma\psi_\pi\right) & -\beta\kappa_p\sigma\psi_{\pi^w} & \sigma\kappa_p\xi_w\psi_{\pi^w} + \xi_p\left(1 + \sigma\psi_x + \kappa_w\sigma\psi_{\pi^w}\right) \\ \kappa_w & \kappa_w\left(\sigma - \beta\sigma\psi_\pi\right) & \beta\left(1 + \kappa_p\sigma\psi_\pi + \sigma\psi_x\right) & -\kappa_w\xi_p\sigma\psi_\pi + \xi_w\left(1 + \kappa_p\sigma\psi_\pi + \sigma\psi_x\right) \\ \kappa_p - \kappa_w & \beta + \sigma\kappa_p + \beta\sigma\psi_x + \kappa_w\left(-\sigma + \beta\left(\sigma\psi_\pi + \sigma\psi_{\pi^w}\right)\right) & -\beta\left(1 + \sigma\psi_x + \kappa_p\left(\sigma\psi_\pi + \sigma\psi_{\pi^w}\right)\right) & \left(1 + \xi_w\right)\left(1 + \kappa_p\sigma\psi_\pi + \sigma\psi_x\right) + \left(\kappa_w + \kappa_p\xi_w\right)\sigma\psi_{\pi^w} + \xi_p\left(1 + \sigma\psi_x + \kappa_w\left(\sigma\psi_\pi + \sigma\psi_{\pi^w}\right)\right) \end{pmatrix}$$

$g = \frac{1}{1+\kappa_p\sigma\psi_\pi+\sigma\psi_x+\kappa_w\psi_{\pi^w}}$.

Matrices for Learning.

$$\widehat{B} = f.\begin{pmatrix} 1 + \xi_p + \xi_w & \sigma + \sigma\xi_p - \beta\sigma\psi_\pi + \xi_w\left(\sigma - \beta\left(\sigma\psi_\pi + \psi_{\pi^w}\right)\right) & -\beta\left(\psi_{\pi^w} + \xi_p\left(\sigma\psi_\pi + \psi_{\pi^w}\right)\right) & 0 \\ \kappa_w\xi_p + \kappa_p\left(1 + \xi_w\right) & \sigma\kappa_p\left(1 + \xi_w\right) + \beta\left(1 + \xi_w\right)\left(1 + \sigma\psi_x\right) + \kappa_w\left(\sigma\xi_p + \beta\psi_{\pi^w}\right) & \beta\left(\xi_p\left(1 + \sigma\psi_x\right) - \kappa_p\psi_{\pi^w}\right) & 0 \\ \kappa_w\left(1 + \xi_p\right) + \kappa_p\xi_w & \kappa_w\left(\sigma + \sigma\xi_p - \beta\sigma\psi_\pi\right) + \xi_w\left(\beta + \sigma\kappa_p + \beta\sigma\psi_x\right) & \beta\left(1 + \xi_p + \kappa_p\sigma\psi_\pi + \sigma\left(1 + \xi_p\right)\psi_x\right) & 0 \\ -\kappa_p + \kappa_w & -\beta + \sigma\kappa_p + \beta\sigma\psi_x + \kappa_w\left(-\sigma + \beta\left(\sigma\psi_\pi + \psi_{\pi^w}\right)\right) & \beta\left(1 + \sigma\psi_x + \kappa_p\left(\sigma\psi_\pi + \psi_{\pi^w}\right)\right) & 0 \end{pmatrix}$$

$$\widehat{\delta} = \begin{pmatrix} 0 & 0 & 0 & \frac{-2\xi_p\sigma\psi_\pi+2\xi_w\psi_{\pi^w}}{\xi_w(1+\xi_w)(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x)} \\ 0 & 0 & 0 & \frac{2(\kappa_p\xi_w\psi_{\pi^w}+\xi_p(1+\sigma\psi_x+\kappa_w\psi_{\pi^w}))}{\xi_w(1+\xi_w)(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x)} \\ 0 & 0 & 0 & -\frac{2(\kappa_w\xi_p\sigma\psi_\pi+\xi_w(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x))}{\xi_w(1+\xi_w)(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x)} \\ 0 & 0 & 0 & \frac{-(-1+\xi_w)(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x)+(\kappa_w-\kappa_p\xi_w)\psi_{\pi^w}-\xi_p(1+\sigma\psi_x+\kappa_w(\sigma\psi_\pi+\psi_{\pi^w}))}{\xi_w(1+\xi_w)(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x)} \end{pmatrix}$$

$$\widehat{\kappa} = f.\begin{pmatrix} \sigma\left(1 + \xi_p + \xi_w\right) & -\kappa_p\left(\left(1 + \xi_w\right)\sigma\psi_\pi + \xi_w\psi_{\pi^w}\right) + \kappa_w\left(\psi_{\pi^w} + \xi_p\left(\sigma\psi_\pi + \psi_{\pi^w}\right)\right) \\ \sigma\left(\kappa_w\xi_p + \kappa_p\left(1 + \xi_w\right)\right) & \left(\kappa_w\xi_p + \kappa_p\left(1 + \xi_w\right)\right)\left(1 + \sigma\psi_x\right) \\ \sigma\left(\kappa_w\left(1 + \xi_p\right) + \kappa_p\xi_w\right) & \left(\kappa_w\left(1 + \xi_p\right) + \kappa_p\xi_w\right)\left(1 + \sigma\psi_x\right) \\ \sigma\left(-\kappa_p + \kappa_w\right) & -\left(\kappa_p - \kappa_w\right)\left(1 + \sigma\psi_x\right) \end{pmatrix}$$

$$f = \frac{1}{(1+\xi_w)(1+\kappa_p\sigma\psi_\pi+\sigma\psi_x)+(\kappa_w+\kappa_p\xi_w)\psi_{\pi^w}+\xi_p(1+\sigma\psi_x+\kappa_w(\sigma\psi_\pi+\psi_{\pi^w}))}$$

Appendix B:

$$B = \begin{pmatrix} 1 - \sigma\psi_x & \sigma - \sigma\psi_\pi & -\sigma\psi_{\pi^w} & 0 \\ \kappa_p\left(1 - \sigma\psi_x\right) & \beta + \kappa_p\left(\sigma - \sigma\psi_\pi\right) & -\kappa_p\sigma\psi_{\pi^w} & \xi_p \\ \kappa_w\left(1 - \sigma\psi_x\right) & \kappa_w\left(\sigma - \sigma\psi_\pi\right) & \beta - \kappa_w\sigma\psi_{\pi^w} & -\xi_w \\ \left(\kappa_p - \kappa_w\right)\left(1 - \sigma\psi_x\right) & \beta + \left(\kappa_p - \kappa_w\right)\left(\sigma - \sigma\psi_\pi\right) & -\beta - \left(\kappa_p - \kappa_w\right)\sigma\psi_{\pi^w} & 1 + \xi_p + \xi_w \end{pmatrix}$$

Matrices for Learning

$$\widehat{B}=\begin{pmatrix} 1-\sigma\psi_x & \sigma-\sigma\psi_\pi & -\sigma\psi_{\pi^w} & 0 \\ \frac{\left(\kappa_w\xi_p+\kappa_p(1+\xi_w)\right)(1-\sigma\psi_x)}{1+\xi_p+\xi_w} & \frac{\beta(1+\xi_w)+\kappa_w\xi_p(\sigma-\sigma\psi_\pi)+\kappa_p(1+\xi_w)(\sigma-\sigma\psi_\pi)}{1+\xi_p+\xi_w} & \frac{-\kappa_p(1+\xi_w)\sigma\psi_{\pi^w}+\xi_p(\beta-\kappa_w\sigma\psi_{\pi^w})}{1+\xi_p+\xi_w} & 0 \\ \frac{\left(\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w\right)(1-\sigma\psi_x)}{1+\xi_p+\xi_w} & \frac{\xi_w\left(\beta+\kappa_p(\sigma-\sigma\psi_\pi)\right)+\kappa_w\left(1+\xi_p\right)(\sigma-\sigma\psi_\pi)}{1+\xi_p+\xi_w} & \frac{\beta\left(1+\xi_p\right)-\left(\kappa_w\left(1+\xi_p\right)+\kappa_p\xi_w\right)\sigma\psi_{\pi^w}}{1+\xi_p+\xi_w} & 0 \\ \frac{\left(\kappa_p-\kappa_w\right)(-1+\sigma\psi_x)}{1+\xi_p+\xi_w} & \frac{-\beta+\kappa_w(\sigma-\sigma\psi_\pi)+\kappa_p(-\sigma+\sigma\psi_\pi)}{1+\xi_p+\xi_w} & \frac{\beta+\kappa_p\sigma\psi_{\pi^w}-\kappa_w\sigma\psi_{\pi^w}}{1+\xi_p+\xi_w} & 0 \end{pmatrix}$$

$$\widehat{\delta}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\xi_p}{1+\xi_p+\xi_w} \\ 0 & 0 & 0 & -\frac{\xi_w}{1+\xi_p+\xi_w} \\ 0 & 0 & 0 & \frac{1}{1+\xi_p+\xi_w} \end{pmatrix}$$

$$\widehat{\kappa}=\begin{pmatrix} \sigma & 0 \\ \frac{\sigma(\kappa_w\xi_p+\kappa_p(1+\xi_w))}{1+\xi_p+\xi_w} & \frac{\kappa_w\xi_p+\kappa_p(1+\xi_w)}{1+\xi_p+\xi_w} \\ \frac{\sigma(\kappa_w(1+\xi_p)+\kappa_p\xi_w)}{1+\xi_p+\xi_w} & \frac{\kappa_w(1+\xi_p)+\kappa_p\xi_w}{1+\xi_p+\xi_w} \\ \frac{\sigma(-\kappa_p+\kappa_w)}{1+\xi_p+\xi_w} & \frac{-\kappa_p+\kappa_w}{1+\xi_p+\xi_w} \end{pmatrix}$$

Appendix C:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \sigma \\ -\kappa_p & 1 & 0 & 0 & 0 \\ -\kappa_w & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 \\ -\psi_x & -\psi_\pi & -\psi_{\pi^w} & 0 & 0 \end{pmatrix}\begin{pmatrix} x_t \\ \pi_t \\ {\pi_t}^w \\ w_{t-1} \\ i_t \end{pmatrix}=\begin{pmatrix} 1 & \sigma & 0 & 0 & 0 \\ 0 & \beta & 0 & \xi_p & 0 \\ 0 & 0 & \beta & -\xi_w & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} \mathrm{E}x_{t+1} \\ \mathrm{E}\pi_{t+1} \\ \mathrm{E}{\pi_{t+1}}^w \\ w_t \\ i_{t+1} \end{pmatrix} \tag{29}$$

$$+\begin{pmatrix} 0 \\ -\xi_p \\ \xi_w \\ 0 \\ 0 \end{pmatrix}{w_t}^n+\begin{pmatrix} \sigma & 0 \\ 0 & \kappa_p \\ 0 & \kappa_w \\ 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} {r_t}^n \\ u_t \end{pmatrix}$$

$$B=h.\begin{pmatrix} 0 & -\beta\psi_\pi & -\beta\psi_{\pi^w} & -\xi_p\psi_\pi+\xi_w\psi_{\pi^w} & 1 \\ 0 & \beta-\frac{\beta\kappa_p\psi_\pi}{\kappa_p\psi_\pi+\psi_x+\kappa_w\psi_{\pi^w}} & -\beta\kappa_p\psi_{\pi^w} & \xi_p\psi_x+\left(\kappa_w\xi_p+\kappa_p\xi_w\right)\psi_{\pi^w} & \kappa_p \\ 0 & -\beta\kappa_w\psi_\pi & \beta\left(\kappa_p\psi_\pi+\psi_x\right) & -\kappa_w\xi_p\psi_\pi+\xi_w\left(\kappa_p\psi_\pi+\psi_x\right) & \kappa_w \\ 0 & \beta\left(\psi_x+\kappa_w\left(\psi_\pi+\psi_{\pi^w}\right)\right) & -\beta\left(\psi_x+\kappa_p\left(\psi_\pi+\psi_{\pi^w}\right)\right) & \left(1+\xi_p+\xi_w\right)\psi_x+\kappa_p\left(\left(1+\xi_w\right)\psi_\pi+\xi_w\psi_{\pi^w}\right)+\kappa_w\left(\psi_{\pi^w}+\xi_p\left(\psi_\pi+\psi_{\pi^w}\right)\right) & \kappa_p-\kappa_w \\ \frac{\kappa_p\psi_\pi+\psi_x+\kappa_w\psi_{\pi^w}}{\sigma} & \kappa_p\psi_\pi+\psi_x+\kappa_w\psi_{\pi^w}+\frac{\beta\psi_\pi}{\sigma} & \frac{\beta\psi_{\pi^w}}{\sigma} & \frac{\xi_p\psi_\pi-\xi_w\psi_{\pi^w}}{\sigma} & -\frac{1}{\sigma} \end{pmatrix}$$

$$h=\frac{1}{\kappa_p\psi_\pi+\psi_x+\kappa_w\psi_{\pi^w}}$$

Matrices for Learning

$$\widehat{B} = \begin{pmatrix} 1 & \sigma & 0 & 0 \\ \frac{\kappa_w \xi_p + \kappa_p(1+\xi_w)}{1+\xi_p+\xi_w} & \frac{\beta + \sigma\kappa_w\xi_p + \beta\xi_w + \sigma\kappa_p(1+\xi_w)}{1+\xi_p+\xi_w} & \frac{\beta\xi_p}{1+\xi_p+\xi_w} & 0 \\ \frac{\kappa_w(1+\xi_p)+\kappa_p\xi_w}{1+\xi_p+\xi_w} & \frac{\sigma\kappa_w(1+\xi_p)+(\beta+\sigma\kappa_p)\xi_w}{1+\xi_p+\xi_w} & \frac{\beta(1+\xi_p)}{1+\xi_p+\xi_w} & 0 \\ \frac{-\kappa_p+\kappa_w}{1+\xi_p+\xi_w} & -\frac{\beta+\sigma\kappa_p-\sigma\kappa_w}{1+\xi_p+\xi_w} & \frac{\beta}{1+\xi_p+\xi_w} & 0 \end{pmatrix}$$

$$\widehat{\delta} = \begin{pmatrix} -\sigma\psi_x & -\sigma\psi_\pi & -\sigma\psi_{\pi^w} & 0 \\ -\frac{(\kappa_w\xi_p+\kappa_p(1+\xi_w))\sigma\psi_x}{1+\xi_p+\xi_w} & -\frac{(\kappa_w\xi_p+\kappa_p(1+\xi_w))\sigma\psi_\pi}{1+\xi_p+\xi_w} & -\frac{(\kappa_w\xi_p+\kappa_p(1+\xi_w))\sigma\psi_{\pi^w}}{1+\xi_p+\xi_w} & \frac{\xi_p}{1+\xi_p+\xi_w} \\ -\frac{(\kappa_w(1+\xi_p)+\kappa_p\xi_w)\sigma\psi_x}{1+\xi_p+\xi_w} & -\frac{(\kappa_w(1+\xi_p)+\kappa_p\xi_w)\sigma\psi_\pi}{1+\xi_p+\xi_w} & -\frac{(\kappa_w(1+\xi_p)+\kappa_p\xi_w)\sigma\psi_{\pi^w}}{1+\xi_p+\xi_w} & -\frac{\xi_w}{1+\xi_p+\xi_w} \\ \frac{(\kappa_p-\kappa_w)\sigma\psi_x}{1+\xi_p+\xi_w} & \frac{(\kappa_p-\kappa_w)\sigma\psi_\pi}{1+\xi_p+\xi_w} & \frac{(\kappa_p-\kappa_w)\sigma\psi_{\pi^w}}{1+\xi_p+\xi_w} & \frac{1}{1+\xi_p+\xi_w} \end{pmatrix}$$

$$\widehat{\kappa} = \begin{pmatrix} \sigma & 0 \\ \frac{\sigma(\kappa_w\xi_p+\kappa_p(1+\xi_w))}{1+\xi_p+\xi_w} & \frac{\kappa_w\xi_p+\kappa_p(1+\xi_w)}{1+\xi_p+\xi_w} \\ \frac{\sigma(\kappa_w(1+\xi_p)+\kappa_p\xi_w)}{1+\xi_p+\xi_w} & \frac{\kappa_w(1+\xi_p)+\kappa_p\xi_w}{1+\xi_p+\xi_w} \\ \frac{\sigma(-\kappa_p+\kappa_w)}{1+\xi_p+\xi_w} & \frac{-\kappa_p+\kappa_w}{1+\xi_p+\xi_w} \end{pmatrix}$$

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